

Add and Subtract Integers



Getting the Idea

The **absolute value** of a number is its distance from 0. For example, the absolute value of 2, written $|2|$, is 2 because it is 2 units to the right of 0 on the number line. Likewise, $|-2|$ is also 2 since -2 is 2 units to the left of 0 on a number line.

You can use a number line to add integers. Start at the point that represents the first integer. To add a positive integer, move to the right. To add a negative integer, move to the left.

Recall that the additive inverse of a number is its opposite. For example, the additive inverse of 5 is -5 .

Example 1

Find the sum of 3 and its additive inverse.

Strategy Use a number line.

Step 1

Write an addition expression for the sum.

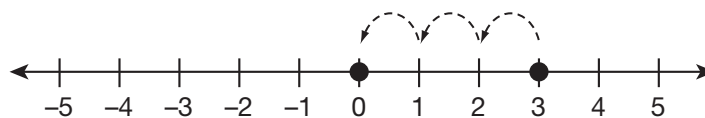
The additive inverse of 3 is -3 .

Find $3 + (-3)$.

Step 2

Use a number line to add.

Start at 3. Since you are adding a negative integer, move 3 units to the left.



The sum is 0.

Solution The sum of 3 and its additive inverse is 0.

$3 + (-3) = 0$ is an example of the existence of the additive inverse property. It states that the sum of a number and its additive inverse is 0.

In Example 1, notice that the sum of $3 + (-3)$ is at 0, located a distance of 3 units to the left of 3. So, $(-3) + 3$ will also have the sum of 0 because it is located 3 units to the right of -3 .

Let a and b represent two integers. To find the sum of $a + b$ on a number line, start at a and move a distance of $|b|$. Move to the right of a if b is positive and to the left of a if b is negative. The sign of the sum depends upon the direction and the number of units moved from a .

Example 2

Find the sum.

$$-4 + 3 = \square$$

Strategy Use a number line to add the two integers.

Start at -4 .

Since you are adding a positive integer, move 3 units to the right.



The sum is -1 .

Solution $-4 + 3 = -1$

You can use the following rules to add integers.

Rules for Adding Two Integers

- When integers have the same sign, add the absolute values and use the sign of the addends in the sum.
- When integers have different signs, find the difference of their absolute values. Then use the sign of the addend with the greater absolute value in the sum.

Example 3

Add.

$$-11 + (-8) = \square$$

Strategy Apply the rules for adding two integers.

Step 1

The integers have the same sign, so add the absolute values.

$$|-11| = 11$$

$$|-8| = 8$$

$$11 + 8 = 19$$

Step 2

Use the sign of the addends.

The sign of both addends is negative, so the sum is -19 .

Solution $-11 + (-8) = -19$

You can also use the properties of addition to add integers.

Example 4

Add.

$$24 + (-10) = \square$$

Strategy Use the properties of addition.

Step 1 Rewrite 24 as a sum with an addend of 10.

$$24 = (14 + 10)$$

Step 2 Rewrite the problem using the new form of 24.

$$24 + (-10) = (14 + 10) + (-10)$$

Step 3 Use the associative property of addition.

$$\begin{aligned}(14 + 10) + (-10) &= 14 + (10 + (-10)) \\ &= 14 + 0 \quad \leftarrow \text{The sum of a number and its} \\ &\quad \text{additive inverse is 0.} \\ &= 14\end{aligned}$$

Solution $24 + (-10) = 14$

A number line can also be used to subtract integers. To subtract a positive integer, move to the left. To subtract a negative integer, move to the right.

Example 5

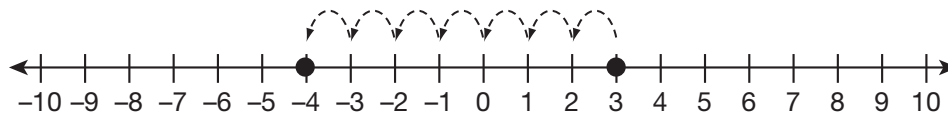
Find the difference.

$$3 - 7 = \square$$

Strategy Use a number line to subtract two integers.

Start at 3.

Since you are subtracting a positive integer, move 7 units to the left.



The difference is -4 .

Solution $3 - 7 = -4$

Subtracting an integer is the same as adding its additive inverse.

Use these rules to subtract integers.

Rules for Subtracting Two Integers

- Write the additive inverse (opposite) of the number to be subtracted (the subtrahend).
- Change the minus sign to a plus sign.
- Apply the rules for adding two integers.

Example 6

Subtract.

$$-5 - 4 = \square$$

Strategy Add the opposite of the subtrahend.

Step 1

Find the opposite of the number to be subtracted.

The subtrahend is 4.

The opposite of 4 is -4 .

Step 2

Add the opposite of the subtrahend to the minuend.

$$-5 - 4 = -5 + (-4)$$

Both integers being added have a negative sign.

Step 3

Add the absolute values of the integers.

$$|-5| = 5 \text{ and } |-4| = 4$$

$$5 + 4 = 9$$

Step 4

Give the sum the same sign as the addends, a negative sign.

Solution $-5 - 4 = -9$

Example 7

Subtract.

$$2 - (-8) = \square$$

Strategy **Add the opposite of the subtrahend.**

Step 1

Find the opposite of the number to be subtracted.

The subtrahend is -8 .

The opposite of -8 is 8 .

Step 2

Add the opposite of the subtrahend to the minuend.

$$2 - (-8) = 2 + 8$$

Both integers being added are positive.

Step 3

Add the integers.

$$2 + 8 = 10$$

Since both integers are positive, the sum will also be positive.

Solution $2 - (-8) = 10$

The properties of addition and subtraction can be used to show that $a - (b + c) = a - b - c$ if a , b , and c are integers.

$$a - (b + c) = a + -(b + c)$$

Add the opposite.

$$= a + (-b) + (-c)$$

Rewrite the sum using the distributive property.

$$= a - b - c$$

Use the properties of subtraction.

You can use the rules for adding and subtracting integers to solve problems.

Example 8

Carly has \$50 in a bank account. She writes a check for \$60 from the account. How much money does Carly have in her account after writing the check?

Strategy **Write a number sentence for the problem. Then solve.**

Step 1

Write a number sentence for the problem.

Let m represent the amount Carly has in her account after writing the check.

$$\$50 - \$60 = m$$

Step 2

Add the opposite of the number to be subtracted.

$$\$50 - \$60 = \$50 + (-\$60)$$

The integers being added have different signs.

Step 3

Find the difference of the absolute values of the integers.

$$|50| = 50$$

$$|-60| = 60$$

$$60 - 50 = 10$$

Step 4

Use the sign of the addend with the greater absolute value.

$|-60| > |50|$, so the sum is negative.

$$\$50 + (-\$60) = -\$10$$

Solution

Carly has $-\$10$ in her account after writing the check.

**Coached Example**

The record low temperature for Albany, New York, was -28°F in January 1971.

The lowest temperature in U.S. history is 52°F lower than Albany's record low temperature. What is the lowest temperature in U.S. history?

Let l represent the lowest temperature in U.S. history.

Write a number sentence to represent the problem. _____

Is the subtrahend positive or negative? _____

Find the opposite of the subtrahend. _____

Add the opposite of the subtrahend to the minuend. _____

Both integers being added have a _____ sign.

Apply the rules for adding two integers.

Find the absolute value of the first addend. _____

Find the absolute value of the second addend. _____

Add the absolute values. _____

Use the sign of the addends in the sum. The sign for the sum is _____.

The lowest temperature in U.S. history is _____ $^{\circ}\text{F}$.



Lesson Practice

Choose the correct answer.

1. Subtract.

$$3 - (-6) = \square$$

- A. -9
- B. -3
- C. 3
- D. 9

2. Add.

$$9 + (-16) = \square$$

- A. 25
- B. 7
- C. -7
- D. -25

3. Subtract.

$$-10 - 4 = \square$$

- A. -14
- B. -6
- C. 6
- D. 14

4. The temperature one morning in Shasta was -12°F . By the afternoon, the temperature had risen 8°F . What was the temperature in the afternoon?

- A. 20°F
- B. 4°F
- C. -4°F
- D. -20°F

5. Find the sum.

$$-4 + (-2) = \square$$

- A. -6
- B. -2
- C. 2
- D. 6

6. Find the difference.

$$6 - 11 = \square$$

- A. -17
- B. -5
- C. 5
- D. 17

7. A submarine at -28 feet dives 40 feet. What is the submarine's elevation after the dive?
- A. 68 feet
 - B. 12 feet
 - C. -12 feet
 - D. -68 feet
8. The Panthers lost 6 yards on their first play and lost another 8 yards on their next play. What was their net result in yards after these two plays?
- A. -14 yards
 - B. -2 yards
 - C. 2 yards
 - D. 14 yards

9. The temperature at the base of a mountain was 14°F . The temperature at the summit was 16°F lower than at the base.

A. Write a subtraction expression to represent the temperature at the summit.

B. What was the temperature at the summit? Show your work.

10. Which word problem has the solution of -4 ? Circle all that apply.
- A. Earl jogged 5 yards forward and then jogged 9 yards backward. What was his final position compared to his starting point?
 - B. A rainbow trout was swimming at -2 feet. It swam downward 4 more feet. What was the new depth of the trout?
 - C. Clarissa had \$49 in her checking account. She spent \$53 on a pair of shoes. What was the new balance of her account?

11. Simplify each expression. Write each expression in the correct box.

$8 + (-2)$	$-6 - 4$	$2 - 4$	$-13 + 3$	$4 - (-2)$
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-10	6	-2

12. A scuba diver is at -4 feet. He dives down 7 more feet to a coral reef. Circle the elevation of the top of the coral reef.

The elevation of the top of the coral reef is

- | |
|-----|
| 11 |
| 3 |
| -3 |
| -11 |

feet.

13. Select True or False for each equation.

- A. $4 - (-6) = 10$ True False
B. $5 + (-11) = -6$ True False
C. $-7 + (-3) = 10$ True False
D. $2 - 9 = -7$ True False

14. Use numbers from the box to complete each equation.

$$-21 + 34 = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} + (-8) = 26$$

$$17 - (-38) = \underline{\hspace{2cm}}$$

$$64 - \underline{\hspace{2cm}} = 30$$

$$-5 - (-18) = \underline{\hspace{2cm}}$$

13

34

55

15. Draw a line from each expression to its equivalent value.

A. $-3 + (-5)$ ● ● 8

B. $14 - 6$ ● ● -8

C. $-5 - 8$ ● ● 13

D. $4 - (-9)$ ● ● -13

16. The temperature at noon was 72°F . The temperature dropped 16°F by 9:00 P.M. Circle the temperature at 9:00 P.M.

The temperature at 9:00 P.M. was 37
56
88 $^{\circ}\text{F}$.